Data Wrangling and Data Analysis

Data Streams

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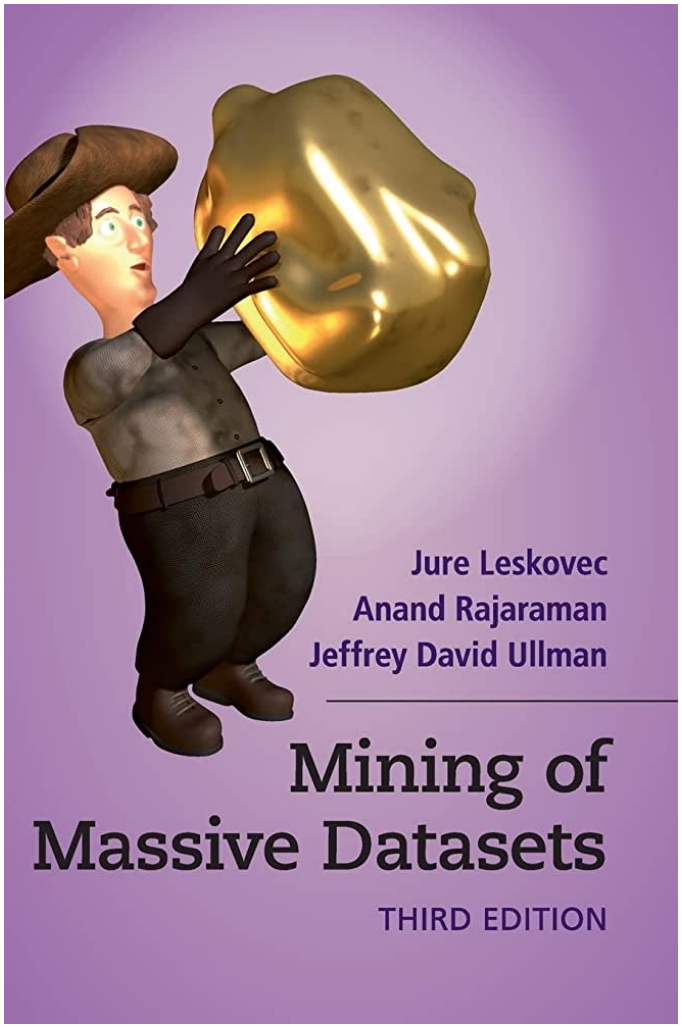
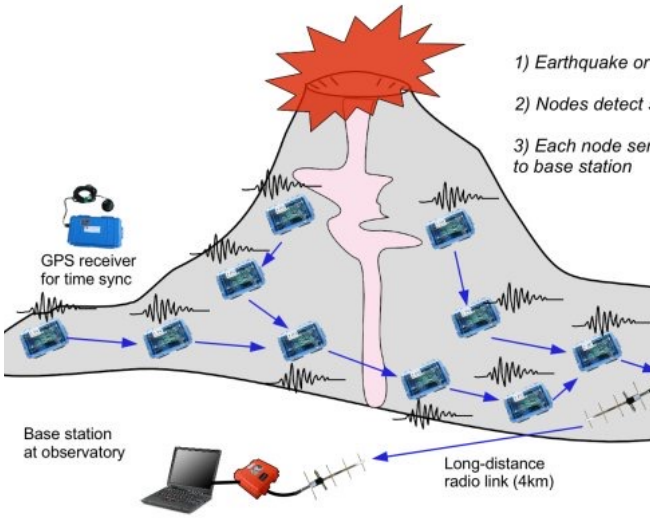
Topics for Today

- Mining Data Streams

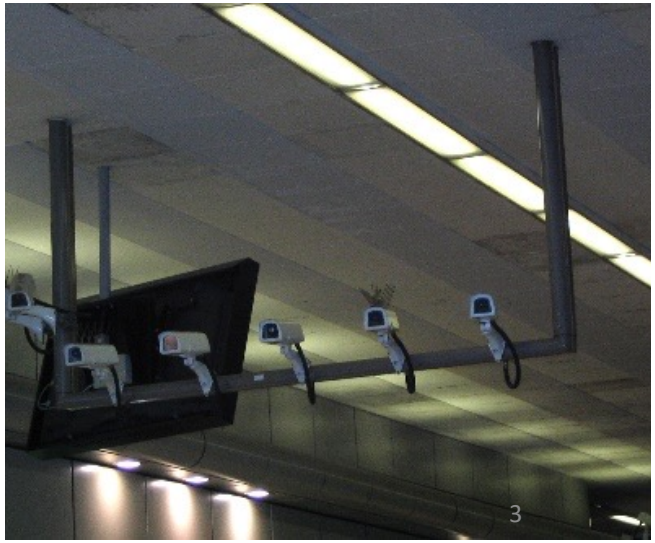
Reading Material

- Mining of Massive Datasets
  - Chapter 4 (4.1 – 4.4)




1) Earthquake or
2) Nodes detect
3) Each node se
to base station
GPS receiver
for time sync
Base station
at observatory
Long-distance
radio link (4km)


GPS
11:58
SW
1 mi
left



Large Hadron Collider
generates 40TB data
per second

A Boeing Jet Engine
creates 20TB
information every Hour

YouTube

YouTube viewers watch
over one billion hours of videos
on its platform every single day

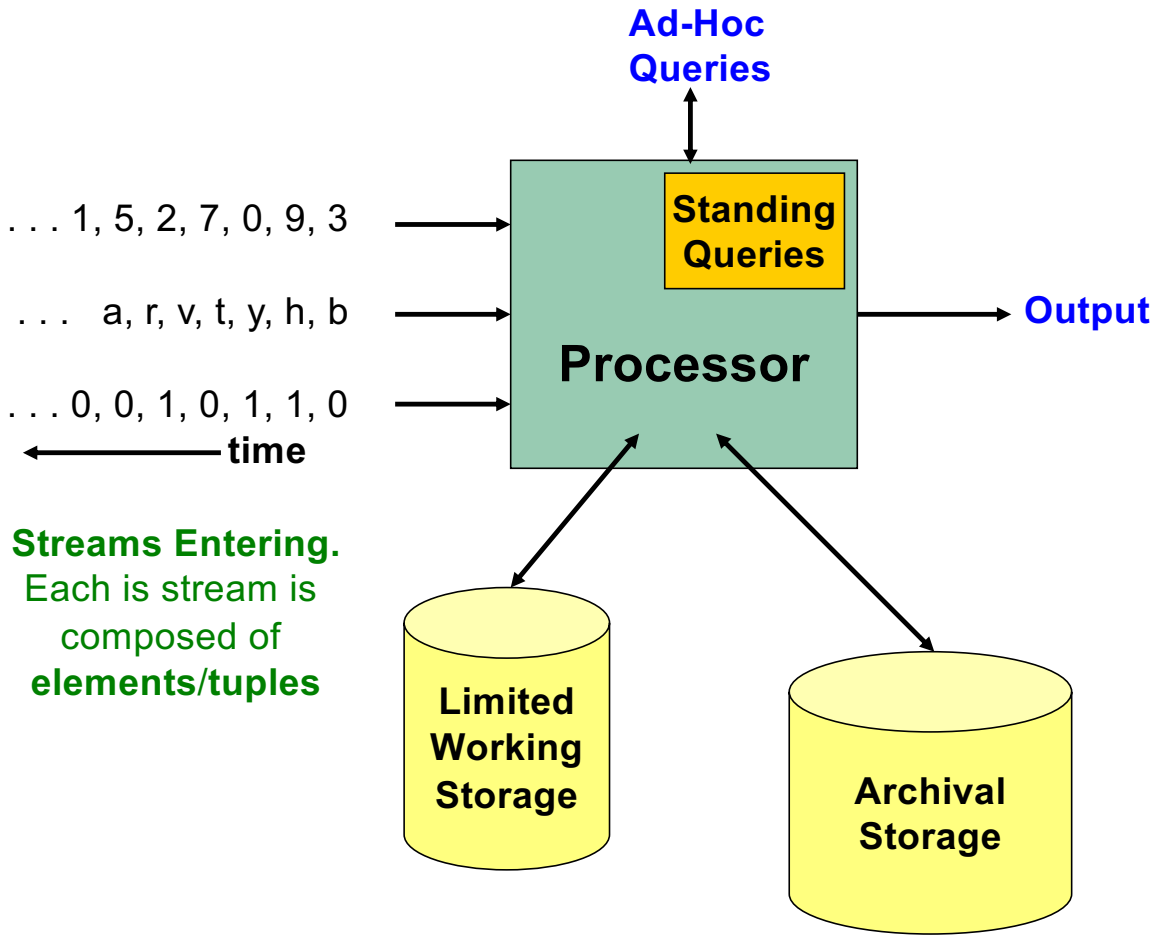
Applications

- Telecommunication calling records
- Business: credit card transaction flows
- Network monitoring and traffic engineering
- Financial market: stock exchange
- Engineering & industrial processes: power supply & manufacturing
- Sensor, monitoring & surveillance: video streams
- Web logs and Web page click streams
- Massive data sets (even saved but random access is too expensive)

Characteristics of Data Streams

- Entire data is not available
- Data arrives (more likely) at a high-speed rate
- The system cannot store the entire stream, but only a small fraction
- Huge volume of continuous data (possibly infinite)
  - Requires single scan algorithms (can only have one look)
- Distribution is non-stationary
  - Requires fast, real-time response

General Stream Processing Model



How can we perform critical calculations on data streams using a limited size of memory?

Handling Data Streams

- Online learning
- Sampling data from data streams
- Windowing functions (models)

Online Learning

- Main idea: perform small changes to update the model
- Training: use the first batch of the data to train a model
- Updating: upon the arrival of a new samples from the stream, slightly update the model

Training

$w_1 \leftarrow 0$

FOR $t = 1$ to T DO

$w_{t+1} \leftarrow w_t - \eta_t l(w_t^\top x_t, y_t)$

Updating

Upon the arrival of new sample

$w_{t+1} \leftarrow w_t - \eta_t l(w_t^\top x_t, y_t)$

- Problem: concept drifts

Sampling Data Stream

Sampling from Data Streams

Since we cannot store the entire stream, one obvious approach is to store a sample

Two different problems:

1. Sample a fixed proportion of elements in the stream (say 1 in 10)
2. Maintain a random sample of fixed size over a potentially infinite stream
   - At any "time" t, we would like a random sample of s elements
     - What are the properties of the sample we want to maintain?

Sampling from Data Streams

Since we cannot store the entire stream, one obvious approach is to store a sample

Two different problems:

1. Sample a fixed proportion of elements in the stream (say 1 in 10)
2. Maintain a random sample of fixed size over a potentially infinite stream
   - At any "time" t, we would like a random sample of s elements
     - Which property of the sample we want to maintain?
       - For all time steps k, each of the k elements seen so far has equal probability of being sampled; OR
       - The sample is representative of the whole stream

Sampling from Data Streams – Sample a fixed proportion

Assume we have space to store 1/10-th of the stream

- Naïve solution:
  - Generate a random integer in [0..9] for each query
  - Store the sample if the integer is 0, otherwise discard
- Problem:
  - As the stream grows, the sample size will grow also

Sampling from Data Streams – Sample a fixed Size sample

- Suppose we need to maintain a random sample S of size exactly s tuples (examples)
  - E.g., main memory size constraint
- Why? Don't know length of stream in advance
- Suppose at time t we have seen n items
  - Each item is in the sample S with equal prob. s/n

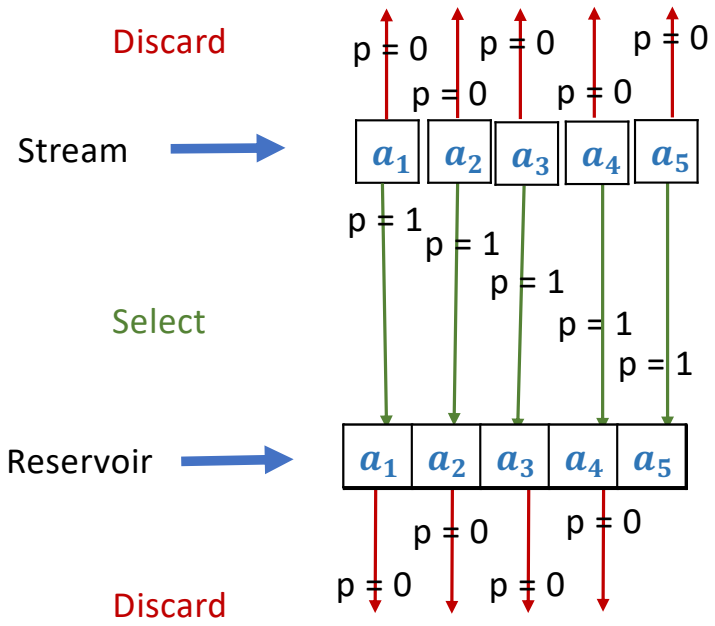
Sampling from Data Streams – Sample a fixed Size sample

- How to think about the problem: say $s = 4$
- Stream: a x c y z k c d e g…
- We need to maintain:
  - When $n = 5$, each of the first 5 tuples is included in the sample S with equal prob.
  - When $n = 7$, each of the first 7 tuples is included in the sample S with equal prob.
- Impractical solution:
  - store the n tuples seen so far
  - pick s at random

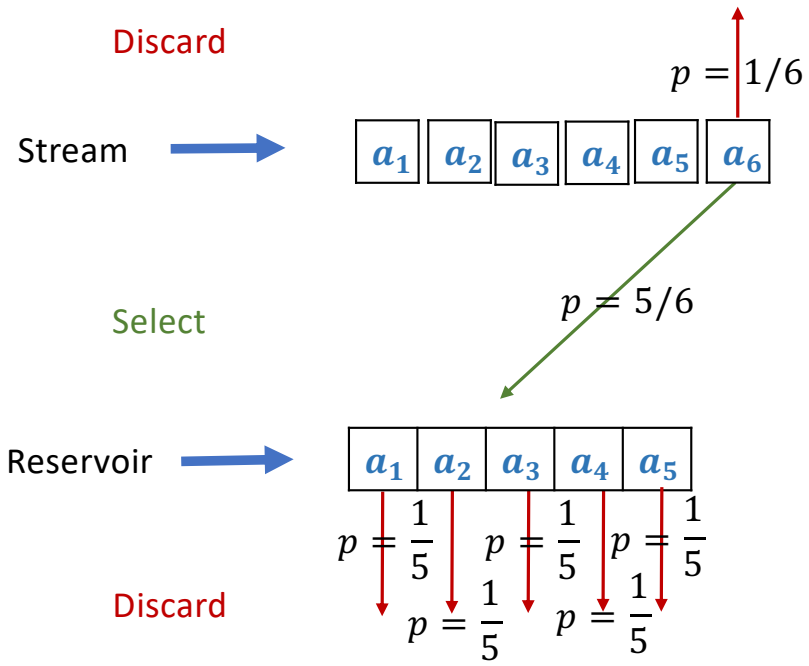
Sampling from Data Streams – Reservoir Sampling

- Store all the first s elements of the stream to S
- Suppose we have seen $n-1$ elements, and now the n-th element arrives ($n > s$)
- With probability s/n, keep the n-th element, else discard it
- If we picked the n-th element, then it replaces one of the s elements in the sample S, picked uniformly at random

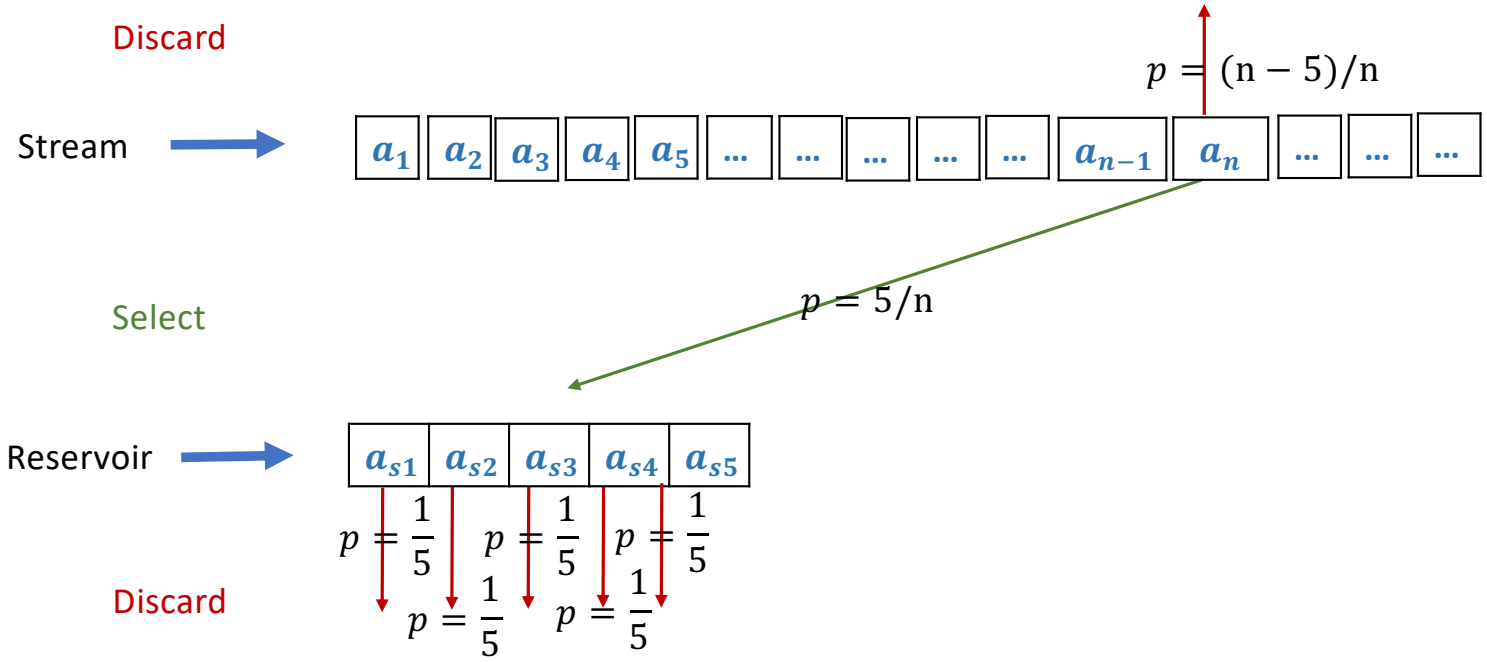
Sampling from Data Streams – Reservoir Sampling



Sampling from Data Streams – Reservoir Sampling



Sampling from Data Streams – Reservoir Sampling

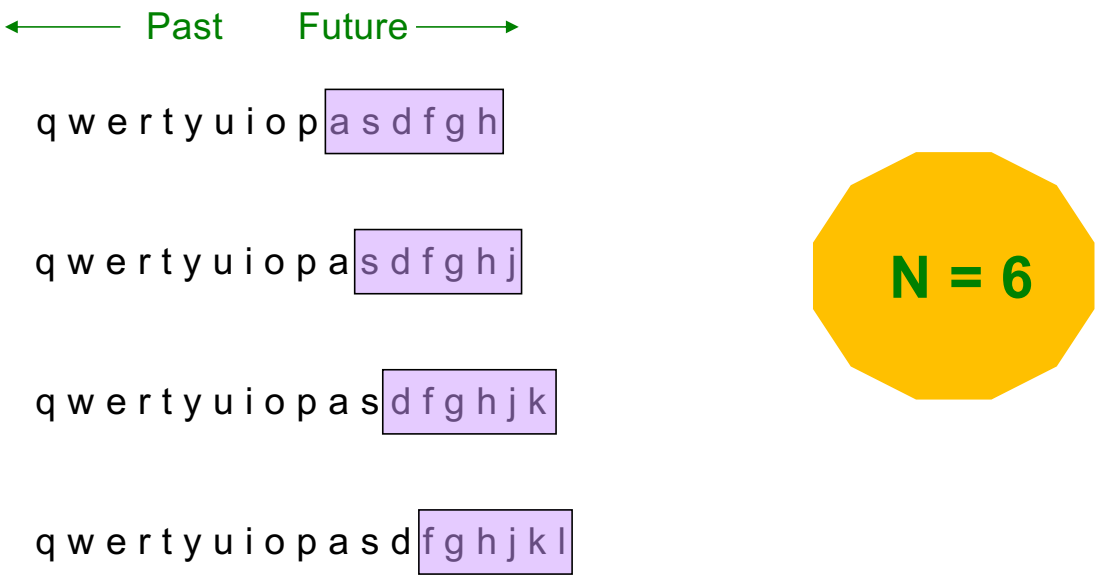


Windowing Models

Windowing Models

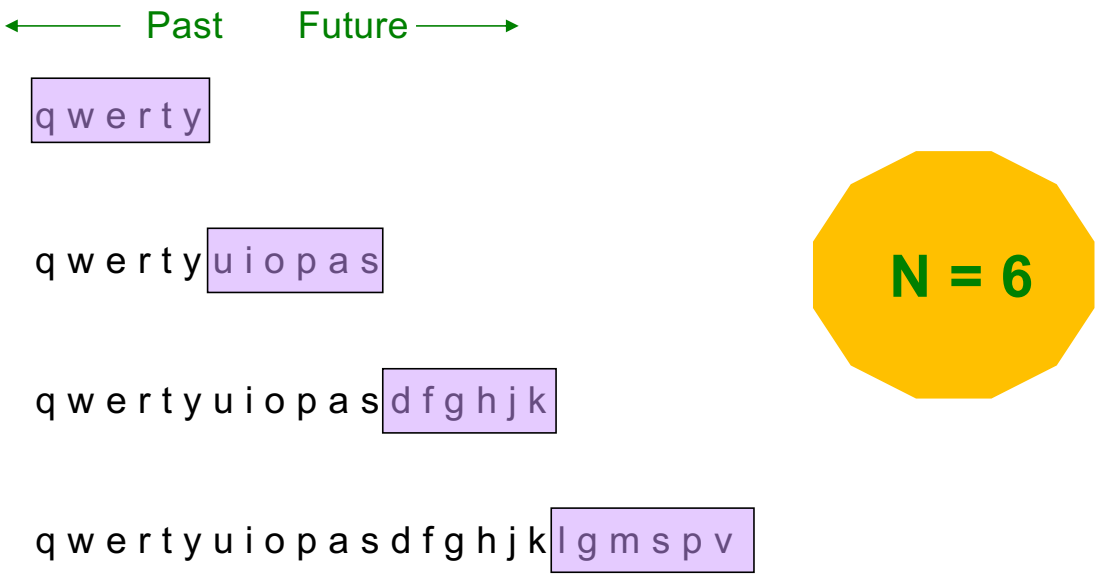
- A useful model of stream processing is that queries are about a window of length N – the N most recent elements received
- Interesting cases:
  - N is so large that the data cannot be stored in memory, or even on disk
  - Or, there are so many streams that windows, for all, cannot be stored
- Amazon example:
  - For every product X we keep 0/1 stream of whether that product was sold in the n-th transaction
  - We want answer queries, how many times have we sold X in the last k sales

Sliding Window



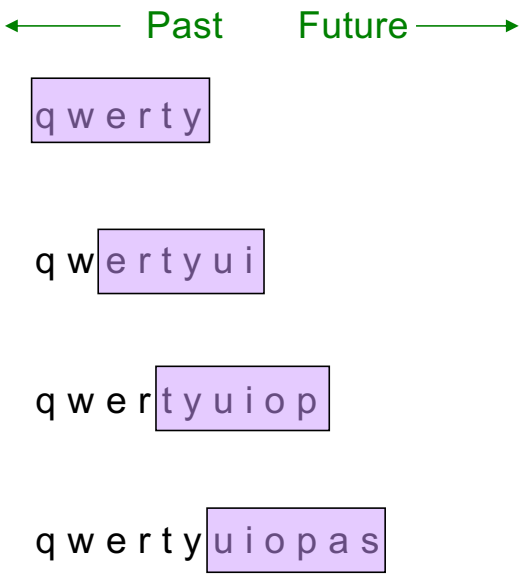
- Upon the arrival of a new item from the stream
  - Discard the oldest item

Tumbling Window (Disjoint Windows)



- Upon the arrival of a new batch of items (of size N) from the stream
  - Discard the previous batch

Hopping Window



N = 6,
Step = 2

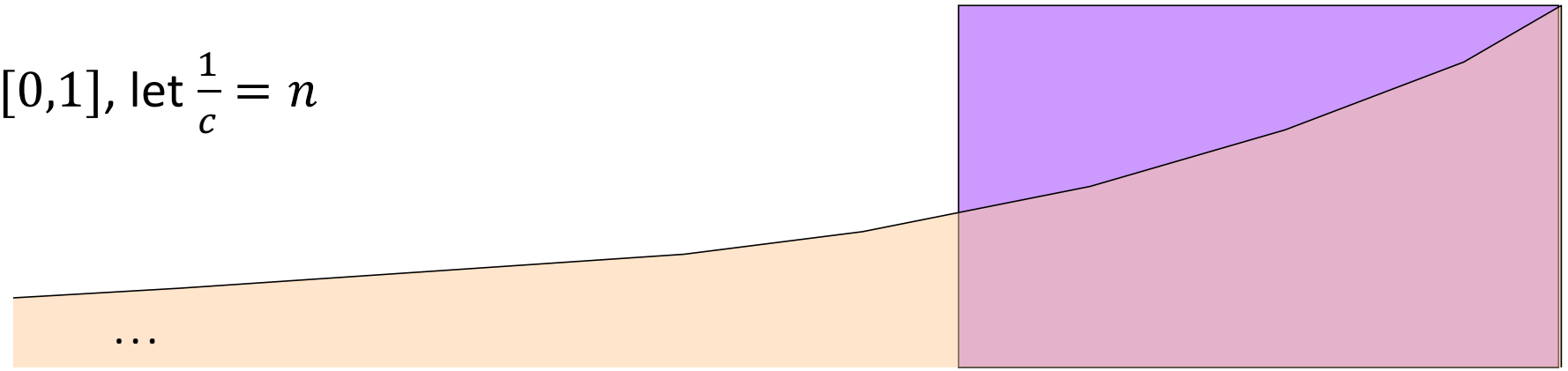
- Upon the arrival of a new (Step) of items from the stream
  - Keep the last N items only

Exponentially Decaying Windows

- Main Idea:
  - Every sample in the stream is important
  - Different levels of importance
  - Recent values are more important
- How it works:
  - Pick a constant $c \in [0,1]$
  - The weight of the element (item) arrived at time t is proportional to $(1-c)^t$
  - $f_t = f(a_t) + (1-c)\sum_{i=1}^{t} f(a_i)$

Sliding vs Exponentially Decaying Windows

- $c \in [0,1]$, let $\frac{1}{c} = n$



- **Important property:**
  - Sum over all weights
    - Sliding window $\sum_{i=t-n+1}^{t} 1 = n = \frac{1}{c}$
    - Exponentially decaying window: $\sum_t (1-c)^t$ is $\frac{\mathbf{1}}{\mathbf{1-(1-c)}} = \frac{\mathbf{1}}{\boldsymbol{c}} = \boldsymbol{n}$

Examples of Queries over Data Streams

Querying Data Streams (Examples)

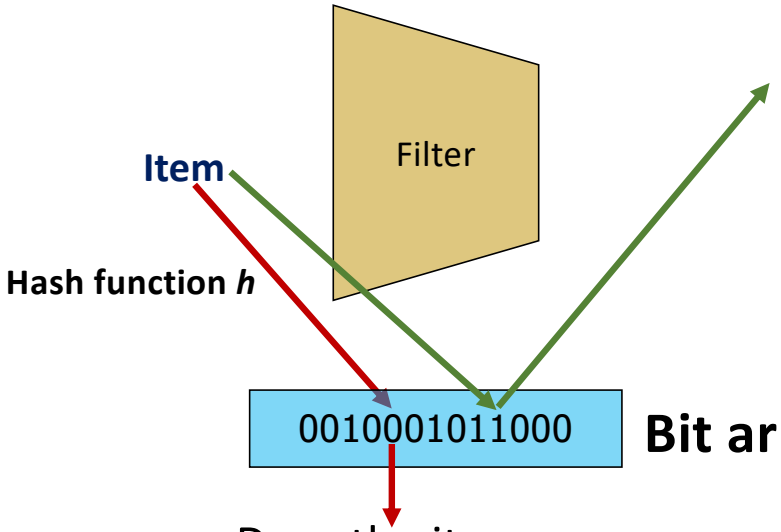
- Filtering a data stream
  - Select elements with property x from the stream
  - Email spam filtering
- Counting distinct elements
  - Number of distinct elements in the last k elements of the stream
  - How many distinct products have we sold in the last week?
- Estimating moments
  - Estimate avg./std. dev. of last k elements
- Finding frequent elements
  - What are “currently” the most popular movies?

Filtering Data Streams

Filtering Data Streams

- Given: a set of Keys *S*
- Determine: which tuples of the stream are in *S*
- Obvious solution: Hash Table
- Problem: we may not have enough memory

Filtering Data Streams



Output the item since it may be in S.
Item hashes to a bucket that at least one of the items in *S* hashed to.

- If the item in *S* (set of keys), return it.
- If the item is not in *S*, it may still be returned (no FNs, but FPs)

Filtering Data Streams (discussion)

- We have:
  - |S| = 1M (We have one million legitimate email addresses)
  - |B| = 1MB (Bit array with 8 million bits)
- Question:
  - What is the probability that an email with un-registered address is going through?

Filtering Data Streams (discussion)

- Approximately 1/8 of the bits will be set to 1
- Given a spam email, it will hash to a bit that includes 1 with $p = 1/8$
- Approximately (1/8 = 0.125) of the spam emails may go through.
  - This is called the false positive ratio (FP ratio)

Filtering Data Streams (discussion)

- More accurate estimation using throwing darts
  - |S| = M, |B| = N
  - Probability of hitting $p_h = \frac{1}{N}$ and missing $p_m = 1 - \frac{1}{N}$
  - After M trials, $p_m = \left(1 - \frac{1}{N}\right)^M$ = $(1-\frac{1}{N})^{N(\frac{M}{N})}$ = $((1-\frac{1}{N})^N)^{(\frac{M}{N})}$
  - $(1-\frac{1}{N})^N \xrightarrow[N\to\infty]{} \frac{1}{e}$
  - Hence $p_m = e^{-\frac{M}{N}}$, $p_h = 1 - e^{-\frac{M}{N}}$
  - M = 1M and N = 8M then $p_h = 1 - e^{-0.125} = 0.1175$ (FP ratio)
- How can we reduce the false positive rate?

Bloom Filters

- We have:
  - |S| = M, |B| = N
- Use k hashing functions $H = h_1, h_2, \ldots, h_k$

$B \leftarrow zeros$

FOR $m \in M$ DO

FOR $h_i \in H$ DO

$B[h_i(m)] \leftarrow 1$

Bloom Filters

- Upon receiving an item x from the stream

$exists \leftarrow 1$

FOR $h_i \in H$ DO

if $B[h_i(x)] == 0$ DO

$exists \leftarrow 0$

Return $(exists)$

- Declare x is in S if the items hashes to a bit with 1 for every hashing function in H

Bloom Filters

- Using the previous analysis
- We have kM trials towards the N targets
  - Fractions of 1s is $(1 - \mathrm{e}^{-\frac{kM}{N}})$
  - Hitting k 1s for the k hashing functions with probability $p_h = \left(1 - \mathrm{e}^{-\frac{kM}{N}}\right)^k$
  - is the probability of a FP

Bloom Filters

- When (|S| = M = 1M and |B| = N = 8M):
  - $k = 1: (1 - e^{-0.125})^1 = 0.1175$
  - $k = 2: (1 - e^{-0.25})^2 = 0.0489$
  - For this example, optimal case when $k = 6: (1 - e^{-0.75})^6 = 0.0216$
  - $k = 7: \left(1 - e^{-7/8}\right)^7 = 0.0229$

Counting Distinct Elements

Counting Distinct Elements

- Problem:
  - Data stream consists of a universe of elements
  - Maintain a count of the number of distinct elements seen so far

- Obvious approach: Maintain the set of elements seen so far
  - That is, keep a hash table of all the distinct elements seen so far

Example Queries

- How many different words are found among the Web pages being crawled at a site?
  - Unusually low or high numbers could indicate artificial pages (spam?)
- How many different Web pages does each customer request in a week?
- How many distinct customers accepted to receive promotional offers during the last month?

Using Small Storage

- Real problem: What if we do not have space to maintain the set of elements seen so far?

- Estimate the count in an unbiased way

- Accept that the count may have a little error, but limit the probability that the error is large

Flajolet-Martin Approach

- Pick a hash function h that maps each of the N elements to at least $\log_2(N)$ bits
- For each stream element a, let $r(a)$ be the number of trailing 0s in $h(a)$
  - $r(a)$ = position of first 1 counting from the right
    - E.g., say $h(a) = 12$, then 12 is 1100 in binary, so $r(a) = 2$
- Record R = the maximum $r(a)$ seen
  - $R = \max_a(r(a))$, over all the items a seen so far
- Estimated number of distinct elements = 2^R

The slides after this point are not included in the final exam

Classification

Classifying Data Streams

- Offline classification
  - train a classifier (model) using labelled examples
  - the model is used to predict the label for unlabelled instances
- Best practices
  - split the labelled dataset into train/validate/test
  - maybe use cross-validation to train accurate model
- Online (streaming) classification
  - no clear separation between train/validate/test sets

Classifying Data Streams

- Restrictions
  - process one instance at a time, and inspect it (at most) once
  - limited time to process each instance
  - limited memory
  - be ready to give predictions at any time
  - adapt to changes (concept drifts)

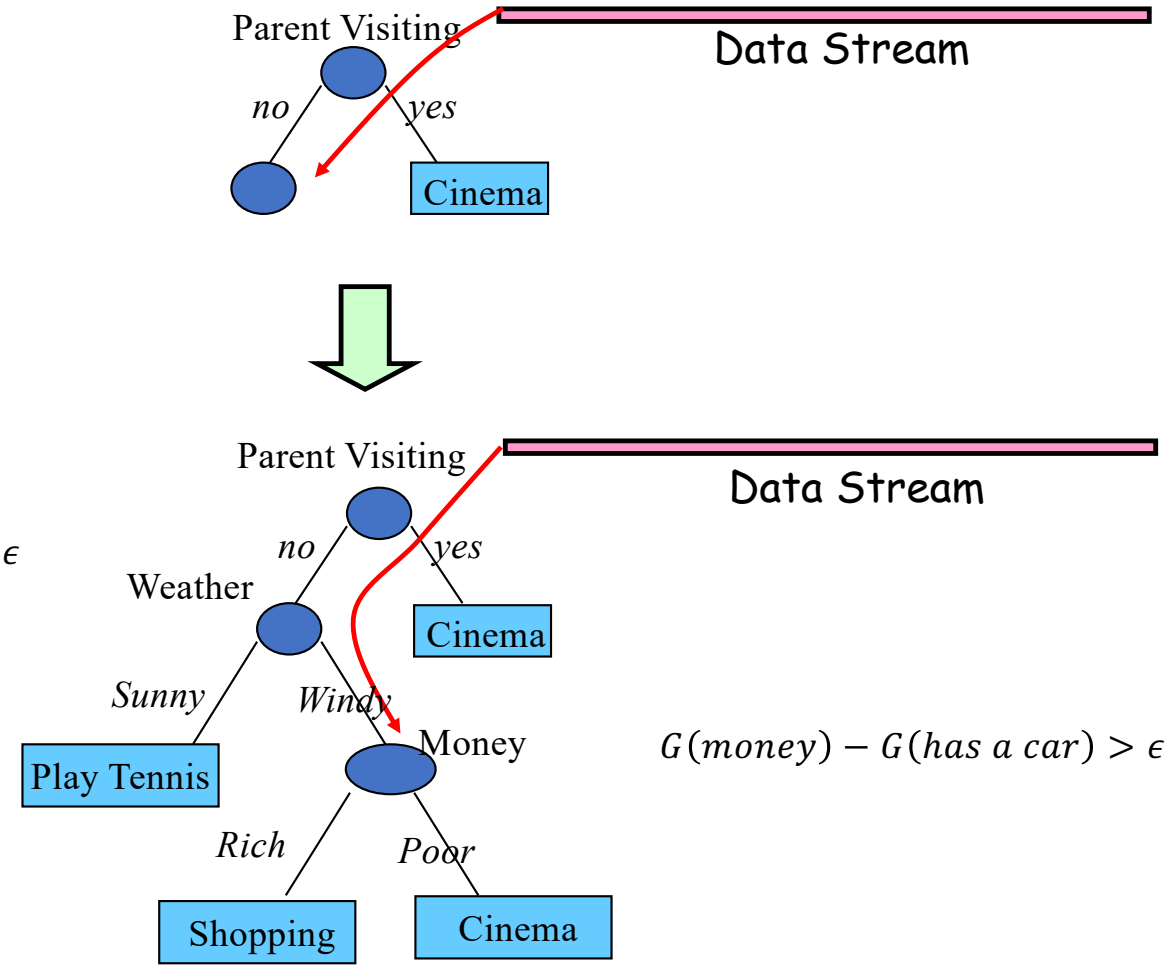
Hoeffding Tree (HT)

- With high probability, HT has similar accuracy as classical DT
- Uses small sample – based on Hoeffding bound
  - X is a random variable
  - R is the domain of X
  - n is the number of observations
  - $\bar{\mu}$ is the sample mean (computed using the n observations)
  - With prob. $1 - \delta$, the distance from μ to $\bar{\mu}$ is at most ϵ, where:

$$\epsilon = \sqrt{\frac{R^2 \ln\left(\frac{1}{\delta}\right)}{2n}}$$

Hoeffding Tree (HT) Algorithm

```
Input:
      S: sequence of observations
      A: set of attributes {A1, A2, …, Am}
      G(.): Attribute Selection Measure
      δ: desired accuracy
Procedure:
      FOR each observation in S:
            compute G(Ai), 1 ≤ i ≤ m
            retrieve Ap, Aq (with two highest G value)
            if (G(Ap) - G(Aq) > ϵ):
                  split on Ap
                  recurse to next node
                  break
```



$G(weather) - G(has\ a\ car) > \epsilon$

$G(money) - G(has\ a\ car) > \epsilon$

HT Strengths and Weaknesses

- Strengths
  - Scales better than traditional methods
  - Incremental: new examples are added as they come
- Weaknesses
  - Could spend a lot of time with ties
  - Memory used with tree expansion
  - Number of candidate attributes

Very Fast Decision Tree (VFDT)

- Modifications to Hoeffding Tree
  - Near-ties broken more aggressively
  - G computed every n_{min} (a user defined parameter)
  - Deactivates certain leaves to save memory
  - Poor attributes dropped
  - Initialize with traditional learner
- Compare to Hoeffding Tree: Better time and memory
- Compare to traditional decision tree
  - Similar accuracy
  - Better runtime with 1.61 million examples
    - 21 minutes for VFDT compared to 24 hours for C4.5

Clustering

Clustering Data Streams

- **Input:** Data stream points from metric space
- **Goal:** Find k clusters in the stream (based on k-median algorithm)
- **Constant factor approximation algorithm**
  - Two step algorithm:
  - Depending on the size of memory, divide the batch of data into l sets $(S_1, \dots, S_l, \quad l \gg k)$
  - Select one center c_i from each S_i, $1 \leq i \leq l$
  - Assign each observation in S_i to its closest center
  - Let $C = \{c_1, \dots, c_l\}$ with each center weighted by number of points assigned to it
  - Cluster C to find k centers (medians)

CluStream

- Divide the clustering process into online and offline components
  - Online component: stores summary statistics about the stream data
    - A micro-cluster for n points is defined as a $(2.d + 3)$ tuple $(\overline{CF2^x}, \overline{CF1^x}, CF2^t, CF1^t, n)$
  - Offline component: answers various user questions based on the stored summary statistics
- Initialization
  - Use the first batch from the stream to cluster the data into q micro-cluster
  - q is significantly larger than the actual number of clusters

CluStream

- Online incremental update of micro-clusters
  - Upon the arrival of a new observation
    - Observation is within max-boundary of one micro-cluster, insert into the micro-cluster
    - Otherwise, create a new cluster
    - May delete obsolete micro-cluster or merge two closest ones
- Query-based macro-clustering (offline)
  - Based on a user-specified time-horizon h and the number of macro-clusters K, compute macro-clusters using the k-medians (or k-means) algorithm

Wrap-Up

- We discussed
  - The data streams
  - Models for handling data streams
  - Example queries over data streams (filtering, counting distinct elements)
  - Classification and Clustering of Data streams

Extra Material for Interested Students

Does each sample have the same probability to be in the reservoir?

Sampling from Data Streams – Reservoir Sampling

- Claim: each element is kept in the reservoir with prob. $p = s/n$
- Proof: We prove that using **mathematical induction**
- Base case:
  - After we see $n = s$ elements, the sample S has the desired property
    - Each sample, (out of $n = s$ elements), is in the sample with probability $s/s = 1$
- Inductive hypothesis:
  - After n elements, the sample S contains each element seen so far with prob. $\frac{s}{n}$
- Now element n+1 arrives

Sampling from Data Streams – Reservoir Sampling

- **Inductive step:** For elements already in S, probability that the algorithm keeps it in S is:

$$\left(1-\frac{s}{n+1}\right)+\left(\frac{s}{n+1}\right)\left(\frac{s-1}{s}\right)=\frac{n}{n+1}$$

Element **n+1** discarded

Element **n+1** not discarded

Element in the sample not picked

- So, at time n, tuples in S were there with prob. s/n
- Time $n \rightarrow n+1$, tuple stayed in S with prob. $n/(n+1)$
- So prob. tuple is in S at time $n+1 = \frac{s}{n} \cdot \frac{n}{n+1} = \frac{s}{n+1}$

Flajolet-Martin Approach for Counting Distinct Items in Data Streams

Why Flajolet-Martin Approach Works?

- Intuition: for a given element a
  - h hashes a to any of the M keys with the same probability
  - h will have a sequence of $\lceil log_2 M \rceil$ bits
  - 2^{-r} is the ratio of the keys that have a tail of 0's
    - Approximately 50% of the keys will hash to ********0
    - Approximately 25% of the keys will hash to ******00
    - Example, if a key hashes to *****100, probably 4 keys were hashed (2^2 keys)

Why Flajolet-Martin Approach Works?

- Formally:
  - We use p_r to be the probability of finding a tail of r zeros and $\tilde{p}_r$ is the probability of finding NO tail of r zeros, show:
  - $p_r \xrightarrow[M \gg 2^r]{} 1$ and $p_r \xrightarrow[M \ll 2^r]{} 0$
  - $M = |S|$ is the number of keys – distinct elements from the steam
- Proof:
  - $h(x)$ hashed the elements uniformly at random
  - $p_r\big(h(x)\big)$ has a tail of r 0's is 2^{-r}
  - $\tilde{p}_r\big(h(x)\big) = 1 - 2^{-r}$ (probability of finding **NO** tail of r 0's)

Why Flajolet-Martin Approach Works?

- After hashing the M keys,
  - $\tilde{p}_r\big(h(x)\big) = (1 - 2^{-r})^M$ (prob. that $h(x)$ has **NO** tail of length r, $\forall x \in S$)
  - $\tilde{p}_r\big(h(x)\big) = (1 - 2^{-r})^M = (1 - 2^{-r})^{2^r (M2^{-r})} \xrightarrow[2^r \to \infty]{} e^{-M2^{-r}}$
  - When $M \ll 2^r$ then $\tilde{p}_r\big(h(x)\big) \to 1$ and $p_r\big(h(x)\big) \to 0$
  - When $M \gg 2^r$ then $\tilde{p}_r\big(h(x)\big) \to 0$ and $p_r\big(h(x)\big) \to 1$