Data Wrangling and Data Analysis

Data Extraction

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Outlines

- So Far
  - Data types
    - Structured, semi-structured and unstructured
  - Data Models
    - Relational model and entity-relationship model
    - Graphs and trees
    - Objects
  - Data model components
    - Data, structure, semantics, and operations
  - Databases vs file systems
  - DDL and DML
  - Creating and modifying tables (relations) in SQL

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Outlines

- Today
  - Operations on databases
    - Relational algebra
    - SQL

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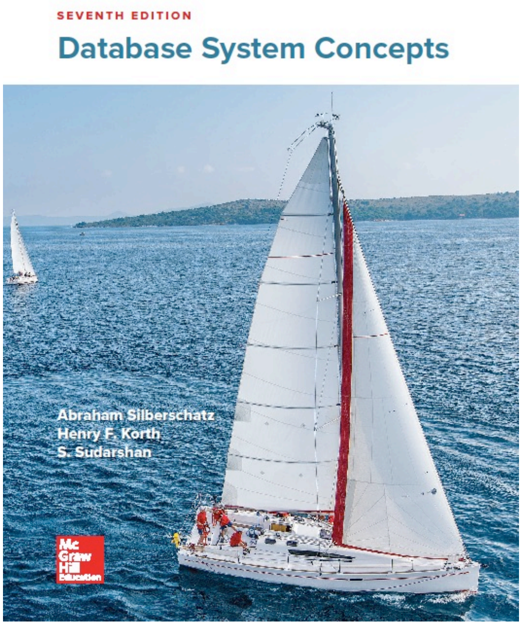
Reading Material for Today

Database System Concepts (7th Edition)

CH 2.6, 3.3 – 3.8.



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Relational Algebra

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Relational Algebra Operators

- Union ∪, intersection ∩, difference −
- Selection σ
- Projection π
- Join ⋈
- Rename ρ

RA

- Duplicate elimination δ
- Grouping and aggregation γ
- Sorting τ

Extended RA

Remember: a relation is a **set** of records

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Union

$r \cup s$

r

A	B
α	3
β	2

s

A	B
α	1
β	2
γ	3

$r \cup s$

A	B
α	1
β	2
γ	3
α	3

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Intersection

$r \cap s$

r

A	B
α	3
β	2

s

A	B
α	1
β	2
γ	3

$r \cap s$

A	B
β	2

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Difference

$$r - s$$

r

A	B
α	3
β	2

s

A	B
α	1
β	2
γ	3

$r - s$

A	B
α	3

Can you find an expression that is equivalent to $r - s$ using the operators $\cap$ and $\sim$?

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Selection

r

A	B	C	D
α	α	1	7
α	β	5	7
β	β	12	3
β	β	23	10

$\sigma_{(A=B)\,\wedge(D>5)}\ (r)$

A	B	C	D
α	α	1	7
β	β	23	10

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Projection

r

A	B	C	D
α	α	1	7
α	β	5	7
β	β	12	3
β	β	23	10

$\pi_{A,C}(r)$

A	C
α	1
α	5
β	12
β	23

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Join

Cartesian Product

r

A	B
α	3
β	2

s

C	D
α	1
β	2
γ	3

$r \times s$

A	B	C	D
α	3	α	1
α	3	β	2
α	3	γ	3
β	2	α	1
β	2	β	2
β	2	γ	3

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Join

Cartesian Product

r

A	B
α	3
β	2

s

A	C
α	1
β	2
γ	3

$r \times s$

r.A	B	s.A	C
α	3	α	1
α	3	β	2
α	3	γ	3
β	2	α	1
β	2	β	2
β	2	γ	3

Rename

Rename operation $\rho_x(E)$ returns the output of the expression E under the name x

r

A	B
α	3
β	2

$r \times \rho_s(r)$

r.A	r.B	s.A	s.B
α	3	α	3
α	3	β	2
β	2	α	3
β	2	β	2

Natural Join

- Let r and s be relations on schemas R and S, respectively.
- Natural join of relations R and S is a relation on schema $R \cup S$ obtained as follows:
  - Consider each pair of tuples t_r from r and t_s from s.
  - If t_r and t_s have the same value on each of the attributes in $R \cap S$, add a tuple t to the result, where
    - t has the same value as t_r on r
    - t has the same value as t_s on s

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Natural Join – Example

r

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

s

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	ϵ

$r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

$$r \bowtie s \equiv \pi_{A,r.B,C,r.D,E}(\sigma_{r.B=s.B \wedge r.D=s.D}\ (r \times s))$$

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Composition of Operations

r

A	B
α	3
β	2

s

A	B
α	1
β	2
γ	3

$r \times s$

r.A	r.B	s.A	s.B
α	3	α	1
α	3	β	2
α	3	γ	3
β	2	α	1
β	2	β	2
β	2	γ	3

$\sigma_{r.A=s.A}\,(r \times s)$

r.A	r.B	s.A	s.B
α	3	α	1
β	2	β	2

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Summary of Relational Algebra Operators

Symbol (name)	Description
σ (Selection)	Return rows of the input relation that satisfy a predicate.
π (Projection)	Return specified attributes from all rows of the input relation. Remove duplicate tuples from the output.
× (Cartesian Product)	Return pairs of rows from the *two* input relations.
∪ (Union)	Return the union of tuples from the *two* input relations.
− (Difference)	Return the set of records that exist in the relation before the operator(-) but not in the relation after the operator.
∩ (Intersection)	Return the common tuples in both input relations.
⋈ (Natural Join)	Return pairs of rows from the *two* input relations that have the same value on all attributes that have the same name.
ρ (Rename)	Returns the outcome of an expression under the specified name

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Remarks on RA

- Each Query input is a table (or set of tables)
- Each query output is a table.
- All data in the output table appears in one of the input tables
- Can we compute:
  - SUM $\gamma_{SUM(tot\_credit)}(student)$
  - AVG $\gamma_{AVG(salary)}(instructor)$
  - MAX $\gamma_{MAX(buget)}(department)$
  - MIN $\gamma_{MIN(budget)}(department)$
  - COUNT $\gamma_{COUNT(tot\_credit \geq 12)}(student)$
  - GROUP BY $_{dept\_name}\,\gamma_{AVG(salary)}(instructor)$

Data Extraction Using Structured Query Language (SQL)

SELECT-FROM-WHERE Statements

SELECT desired attributes

FROM one or more tables

WHERE condition about tuples of the tables

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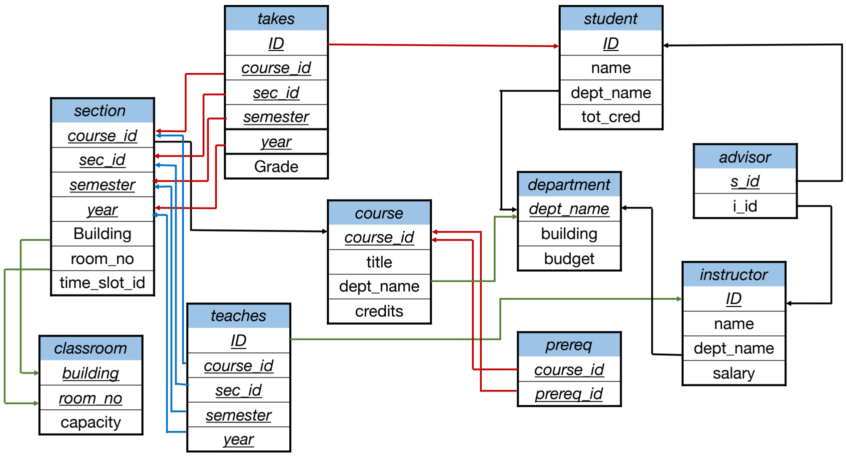
Remember Our Main Database?



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Example: The SELECT Clause

- Get the IDs, names and total credits of students who completed at least 24 credits

```
SELECT ID, name, tot_cred
FROM student
WHERE tot_cred >= 24;
```

- The answer is a relation with three attributes (ID, name, tot_cred)
- What if we need only the names of the students?
- Note that: the SQL operator names are case insensitive SELECT ≡ Select ≡ select

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The SELECT Clause

- Lists the desired attributes in the result of the query
  - Corresponds to the projection operator of the relational algebra
- SQL allows duplicates in relations as well as in query results
- To force the elimination of duplicates, use the keyword **DISTINCT** after select
- Example: find the department names of all instructors whose salary is strictly greater than 60000 without showing the department name more than once

```
SELECT DISTINCT dept_name
FROM instructor
WHERE salary > 60000
```

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The SELECT Clause (Cont.)

- The keyword **ALL** specifies that duplicates should not be removed

```
SELECT ALL dept_name
FROM instructor
WHERE salary > 60000
```

- An asterisk in the **SELECT** clause denotes "all attributes"

  SELECT * FROM instructor

  will return all the records from table "instructor"

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The SELECT Clause (Cont.)

- An attribute could be literal with no **FROM** clause

  SELECT '542'

  Results in a relation with one column and one row with value ''542'

  We can also give the column a name using

  SELECT '542' AS V1
- An attribute could be a literal with **FROM** clause

  SELECT 'A' FROM instructor

  will return a relation with one column and N rows (the number of tuples in the 'instructor ' relation) where each row will contain the value "A"

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The SELECT Clause (Cont.)

- **SELECT** clause can contain arithmetic expressions involving the operations *, +, -, and /.

  SELECT ID, name, salary/12.0

  FROM instructor

  This query would return a relation with the same number of records as the 'instructor' relation and the (ID, name, salary/12.0) where the yearly salary is replaced by the monthly salary

  We can rename the "salary/12.0" using the **AS** clause

  SELECT ID, name, salary/12.0 as monthly_salary

  FROM instructor

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The WHERE Clause

- Specifies conditions that the result must satisfy
  - Corresponds to the selection predicate of the relational algebra
- To find all students enrolled in the 'Math.' department

  SELECT ID, name FROM student

  WHERE dept_name = 'Math.'
- Conditions can be also combined using logical operators (AND, OR, NOT)
  - Find all students in the 'Math.' department who completed a minimum of 24 credits

    SELECT ID, name FROM student

    WHERE dept_name = 'Math.' AND tot_cred >= 24
- Comparisons =, <>, <, >, <=, >=

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The FROM Clause

- Lists the relations involved in the query
  - Corresponds to the Cartesian product operation of the relational algebra
- Find the Cartesian product *' instructor' X ' teaches'*

  SELECT * FROM instructor, teaches
  - Generates every possible instructor – teaches pair, with all attributes from both relations.
  - For common attributes (e.g., *ID*), the attributes in the resulting table are renamed using the relation name (e.g., *instructor.ID*)
  - Similar to the cartesian product in RA
- Cartesian product not very useful directly, but useful when combined with where-clause condition.

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SELECT-FROM-WHERE Examples

SELECT name, course_id

FROM instructor, teaches

WHERE instructor.ID = teaches.ID

Returns the names of all instructors who have taught any courses and the course_id

SELECT name, course_id

FROM instructor, teaches

WHERE instructor.ID = teaches.ID AND instructor.dept_name = 'Art'

Returns the names of all instructors in the Art department who have taught any courses and the course_id

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The Rename Operation

- SQL allows renaming relations and attributes using the **AS** clause:

  old-name **AS** *new-name*

- Find the names of all instructors who have a higher salary than some instructor in 'Comp. Sci'.

```
SELECT DISTINCT T.name
FROM instructor AS T, instructor AS S
WHERE T.salary >= 75000 AND S.dept_name = 'Comp. Sci.'
```

  Returns the names of instructors in the 'Comp. Sci.' department joined with whose salary is greater than or equal 75000

- Keyword **AS** is optional and may be omitted

  instructor **AS** *T ≡ instructor T*

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Renaming Example: Self-Join

- Sometimes, a query needs to use two copies of the same relation.
- Distinguish copies by renaming the relations.
- Example self-Join

```
SELECT T.name AS N1, S.name AS N2
FROM instructor T, instructor S
WHERE T.salary = S.salary AND T.name < S.name
```

  - Returns the names of instructors who has the same salary
  - Do not produce pairs like (Miller, Miller)
  - Produces pairs in alphabetic order, e.g. (Adison, Miller), not (Miller, Adison)
- Note that we omit AS when renaming the relations

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Let us try:
Running a few SQL queries on different platforms (SQLite3 command line, Google Colab, and w3school)

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What have we learnt?

Let us spend 2 minutes to look at what we have studied so far

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Join Operation

- **JOIN** operations take two relations and return as a result another relation.
- A join operation is a Cartesian product which requires that tuples in the two relations match (under some condition). It also specifies the attributes that are present in the result of the join
- The join operations are typically used as subquery expressions in the **FROM** clause

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Join Operation (Cont.)

- We will consider the following relations in the few coming slides
- Relation *course*

course_id	title	dept_name	credits
BIO-301	Genetics	Biology	4
CS-490	Game Design	Comp. Sci.	4
CS-315	Boolean Algebra	Comp. Sci.	3

- Relation *prereq*

course_id	prereq_id
BIO-301	BIO-101
CS-490	CS-101
CS-347	CS-201

- Note that:
  - *prereq* information is missing for course CS-315
  - *course information is missing for course CS-347*

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Outer Join

- An extension of the join operation that avoids loss of information.
- Computes the join and then adds tuples form one relation that does not match tuples in the other relation to the result of the join.
- Uses *null* values.

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Left Outer Join

```
SELECT *
FROM course
LEFT OUTER JOIN prereq
ON course.course_id = prereq.course_id
```

course_id	title	dept_name	credits	prereq_id
BIO-301	Genetics	Biology	4	BIO-101
CS-490	Game Design	Comp. Sci.	4	CS-101
CS-315	Boolean Algebra	Comp. Sci.	3	*null*

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Right Outer Join

SELECT *
FROM *course*
RIGHT OUTER JOIN prereq
ON course.course_id = prereq.course_id

course_id	prereq_id	title	dept_name	credits
BIO-301	BIO-101	Genetics	Biology	4
CS-490	CS-101	Game Design	Comp. Sci.	4
CS-347	CS-201	*null*	*null*	*null*

- Remember:
  - The order of the attributes in a relation has no meaning

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Full Outer Join

SELECT *
FROM *course*
FULL OUTER JOIN prereq
ON course.course_id = prereq.course_id

course_id	prereq_id	title	dept_name	credits
BIO-301	BIO-101	Genetics	Biology	4
CS-490	CS-101	Game Design	Comp. Sci.	4
CS-347	CS-201	*null*	*null*	*null*
CS-315	*null*	Boolean Algebra	Comp. Sci.	3

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Inner Join

SELECT *
FROM *course*
INNER JOIN prereq
ON course.course_id = prereq.course_id

course_id	prereq_id	title	dept_name	credits
BIO-301	BIO-101	Genetics	Biology	4
CS-490	CS-101	Game Design	Comp. Sci.	4

- An SQL INNER JOIN is same as JOIN clause
- Question:
  - What is the difference between the above JOIN and the right/left outer join

String Operations

- SQL includes a string-matching operator for comparisons on character strings.
- The operator **LIKE** uses patterns that are described using two special characters
  - Percent (%). The % character matches any substring.
  - Underscore (_). The _ character matches any character.
- Example: find the names of all instructors whose name includes the substring “Van der”

  SELECT DISTINCT *name*
  FROM *instructor* WHERE *name* LIKE ‘%Van der%’

String Operations (Cont.)

- Match the string “100%”

  LIKE ‘100 \%’ **ESCAPE** '\'

  in that above we use backslash (\) as the escape character
- Patterns are case sensitive.
- Pattern matching examples:
  - ‘Intro%’ matches any string beginning with “Intro”.
  - ‘%Comp%’ matches any string containing “Comp” as a substring.
  - ‘_ _ _’ matches any string of exactly three characters.
  - ‘_ _ _ %’ matches any string of at least three characters.

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Range Queries

- SQL includes a **BETWEEN** comparison operator
- Example: Find the names of all instructors with salary between $90,000 and $100,000 (that is, ≥ $90,000 and ≤ $100,000)

  SELECT *name*
  FROM *instructor*
  WHERE *salary* BETWEEN 90000 AND 100000

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Tuple Comparison

SELECT *name, course_id*
FROM *instructor, teaches*
WHERE *(instructor.ID, dept_name) = (teaches.ID, 'Biology')*

Set Operations

- Find courses that ran in Fall 2009 or in Spring 2010
  SELECT *course_id* FROM *section* WHERE *sem = 'Fall'* AND *year = 2009*
  UNION
  SELECT *course_id* FROM *section* WHERE *sem = 'Spring'* AND *year = 2010*
- Find courses that ran in Fall 2009 and in Spring 2010
  SELECT *course_id* FROM *section* WHERE *sem = 'Fall'* AND *year = 2009*
  INTERSECT
  SELECT *course_id* FROM *section* WHERE *sem = 'Spring'* AND *year = 2010*
- Find courses that ran in Fall 2009 but not in Spring 2010
  SELECT *course_id* FROM *section* WHERE *sem = 'Fall'* AND *year = 2009*
  EXCEPT
  SELECT *course_id* FROM *section* WHERE *sem = 'Spring'* AND *year = 2010*

Null Values

- It is possible for tuples to have a null value, denoted by *NULL*, for some of their attributes
- *null* signifies an unknown value or that a value does not exist.
- The result of any arithmetic expression involving *NULL* is *NULL*
  - Example: 5 + *NULL* returns NULL
- The predicate **IS NULL** can be used to check for null values.
  - Example: Find all instructors whose salary is null.

```
SELECT name
FROM instructor
WHERE salary IS NULL
```

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Null Values and Three Valued Logic

∧	*T*	*F*	*U*
T	T	F	U
F	F	F	F
U	U	F	U

∨	*T*	*F*	*U*
T	T	T	T
F	T	F	U
U	T	U	U

- Three values – *true*, *false*, *unknown*
- Any comparison with *null* returns *unknown*
  - Example: *5 < null or null <> null or null = null*
- Three-valued logic using the value *unknown*:
  - OR: (*unknown* **OR** *true*) = *true*,
    (*unknown* **OR** *false*) = *unknown*
    (*unknown* **OR** *unknown*) = *unknown*
  - AND: *(true* **AND** *unknown)* = *unknown,*
    (false **AND** *unknown)* = *false,*
    (unknown **AND** *unknown)* = *unknown*
  - NOT*:* *(***NOT** *unknown)* = *unknown*
  - "*P* **is unknown**" evaluates to true if predicate *P* evaluates to *unknown*
- Result of **WHERE** clause predicate is treated as *false* if it evaluates to *unknown*

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The IN Operator

- <v> **IN** <S> evaluates to true if the value v matches one of the values in S.
- It can be used to replace a sequence of conditions connected by **OR**
- **Example:**

```
SELECT name
FROM instructor
WHERE dept_name IN ('Comp. Sci.', 'Math.', 'Chem.');
```

This Query is equivalent to:

```
SELECT name
FROM instructor
WHERE dept_name = 'Comp. Sci.' OR dept_name = 'Math.' OR dept_name =
'Chem.'
```


Aggregate Functions

- These functions operate on the multiset of values of a column of a relation, and return a value

avg: average value
min: minimum value
max: maximum value
sum: sum of values
count: number of values

Aggregate Functions (Cont.)

- Find the average salary of instructors in the Computer Science department

```
SELECT AVG (salary)
FROM instructor
WHERE dept_name= 'Comp. Sci.';
```

- Find the total number of instructors who taught a course in the Spring 2010 semester

```
SELECT COUNT (DISTINCT ID)
FROM teaches
WHERE semester = 'Spring' AND year = 2010;
```

- Find the number of tuples in the *course* relation

```
SELECT COUNT (*)
FROM course;
```

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Aggregate Functions (Cont.)

- Find the average salary of instructors in each department

```
SELECT dept_name, AVG (salary) AS avg_salary
FROM instructor
GROUP BY dept_name;
```

ID	name	dept_name	salary
22322	Einstein	Physics	95000
33452	Gold	Physics	87000
21212	Wu	Finance	90000
10101	Brandt	Comp. Sci.	82000
43521	Katz	Comp. Sci.	75000
98531	Kim	Biology	78000
58763	Crick	Elec. Eng.	80000
52187	Mozart	History	65000
32343	El Said	History	86000

The query result

dept_name	avg_salary
Physics	91000
Finance	90000
Comp. Sci.	78500
Biology	78000
Elec. Eng.	80000
History	75500

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Aggregate Functions (Cont.)

- Find the average salary of instructors in each department which has average salary greater than 80000 – use HAVING because WHERE cannot be used with aggregate functions

  SELECT *dept_name*, AVG (*salary*) AS *avg_salary*
  FROM *instructor* GROUP BY *dept_name*
  HAVING avg_salary > 80000;

ID	name	dept_name	salary
22322	Einstein	Physics	95000
33452	Gold	Physics	87000
21212	Wu	Finance	90000
10101	Brandt	Comp. Sci.	82000
43521	Katz	Comp. Sci.	75000
98531	Kim	Biology	78000
58763	Crick	Elec. Eng.	80000
52187	Mozart	History	65000
32343	El Said	History	86000

The query result

dept_name	avg_salary
Physics	91000
Finance	90000

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Subqueries

- SQL provides a mechanism for the nesting of subqueries.
- A **subquery** is a **SELECT-FROM-WHERE** expression that is nested within another query.
- The nesting can be done in the following SQL query

  SELECT $A_1, A_2, \ldots, A_n$
  FROM $r_1, r_2, \ldots, r_n$
  WHERE *P*

  as follows:
  - A_i can be replaced be a subquery that generates a single value.
  - r_i can be replaced by any valid subquery
  - *P* can be replaced with an expression of the form:
    B <operation> (subquery)

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Subqueries – Examples (Subquery in the WHERE clause)

- Find courses offered in Fall 2009 and in Spring 2010

```
SELECT DISTINCT course_id
FROM section
WHERE semester = ’Fall’ AND year = 2009 AND
  course_id IN (SELECT course_id
                FROM section
                WHERE semester = ’Spring’ AND year= 2010);
```

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Subqueries – Examples (Subquery in the FROM clause)

- Find the average instructors’ salaries of those departments where the average salary is greater than $42,000.”

```
SELECT dept_name, avg_salary
FROM (SELECT dept_name, AVG (salary) AS avg_salary
        FROM instructor
        GROUP BY dept_name)
WHERE avg_salary > 42000;
```

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Subqueries – Examples (Subquery in the SELECT clause)

- List all departments along with the number of instructors in each department

```
SELECT dept_name,
          (SELECT COUNT(*)
             FROM instructor
            WHERE department.dept_name = instructor.dept_name)
          AS num_instructors
FROM department;
```

- Runtime error if subquery returns more than one result tuple
- Note that: subqueries are parenthesized SELECT-FROM-WHERE statements

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Wrap-Up

- Summarize what you learned today in 2-minutes

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Boolean Operators – Revision

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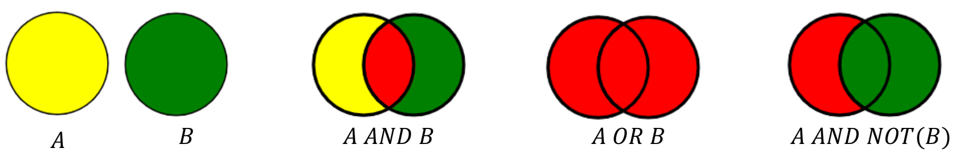
Boolean Operators

- Searching through a database or search engine can often be frustrating
- Boolean Operators create relationships between concepts and terms for better search results
- Most popular Boolean operators are **AND**, **OR** and **NOT**
  - The red areas represent the results of the operators



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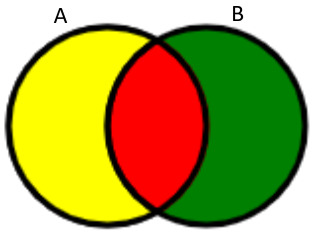
AND (∧)

A	B	$A \wedge B$
1	1	1
1	0	0
0	1	0
0	0	0



The red area – $(A \wedge B)$

- Retrieves only records that satisfy both conditions
- Example:

  name = “Taylor” AND dept_name = “Chem.”

  Returns all instructors in the Chem. department whose name is Taylor

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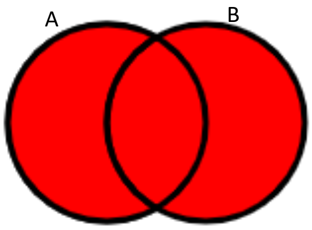
OR (∨)

A	B	$A \vee B$
1	1	1
1	0	1
0	1	1
0	0	0



The red area – $(A \vee B)$

- Retrieves records that satisfy one of the conditions
- Example:

  name = “Taylor” OR dept_name = “Chem.”

  Returns all instructors with the name Taylor and all instructors of the Chem. department

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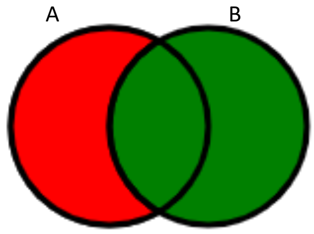
NOT (~)

A	$\sim A$
1	0
0	1



The red area – $A\ AND\ NOT(B)$

- Retrieves only records that satisfy the first condition and doesn't satisfy the second
- Example:

  name = "Taylor" AND

  NOT dept_name = "Chem."

  Returns all instructors with the name Taylor who do not work in the Chem. department

Boolean Equivalence

- Equivalence of two Boolean operations can be easily proven using truth tables
- Equivalence of Boolean operations is useful for optimizing the Boolean queries
- Examples:
  - $\sim (\boldsymbol{A} \wedge \boldsymbol{B}) \equiv \sim \boldsymbol{A} \vee \sim \boldsymbol{B}$ →

A	B	$(A \wedge B)$	$\sim (A \wedge B)$	$\sim A \vee \sim B$
1	1	1	0	0
1	0	0	1	1
0	1	0	1	1
0	0	0	1	1

≡

  - $(\boldsymbol{A} \wedge \boldsymbol{B}) \wedge \sim \boldsymbol{B} \equiv \boldsymbol{A} \wedge (\boldsymbol{B} \wedge \sim \boldsymbol{B}) \equiv \boldsymbol{false}$
  - $(\boldsymbol{A} \wedge \boldsymbol{B}) \vee (\sim \boldsymbol{B} \wedge \boldsymbol{C}) \vee (\boldsymbol{A} \wedge \boldsymbol{C}) \vee \sim (\boldsymbol{A} \wedge \boldsymbol{C}) \equiv (\boldsymbol{A} \wedge \boldsymbol{C}) \vee \sim (\boldsymbol{A} \wedge \boldsymbol{C}) \equiv \boldsymbol{true}$

Proving Boolean Equivalence

- Truth Table: Helpful when the number of statements in the Boolean expression is small
- Proof by contradiction: assume the expression is false/true and proof that it leads to contradiction
- Using the Boolean axioms.

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Boolean Axioms

- Let $\boldsymbol{T}$ and $\boldsymbol{F}$ represent the cases of always True and always False
  - $(\boldsymbol{T} \wedge \boldsymbol{T}) = \boldsymbol{T}$
  - $(\boldsymbol{F} \wedge \boldsymbol{F}) = \boldsymbol{F}$
  - $(\boldsymbol{T} \vee \boldsymbol{T}) = \boldsymbol{T}$
  - $(\boldsymbol{F} \vee \boldsymbol{F}) = \boldsymbol{F}$
  - $(\boldsymbol{T} \wedge \boldsymbol{F}) = (\boldsymbol{F} \wedge \boldsymbol{T}) = \boldsymbol{F}$
  - $(\boldsymbol{T} \vee \boldsymbol{F}) = (\boldsymbol{F} \vee \boldsymbol{T}) = \boldsymbol{T}$
  - $\overline{\boldsymbol{T}} = \boldsymbol{F}$
  - $\overline{\boldsymbol{F}} = \boldsymbol{T}$

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Boolean Axioms

- Commutativity
  - $(\boldsymbol{A} \wedge \boldsymbol{B}) = (\boldsymbol{B} \wedge \boldsymbol{A})$
  - $(\boldsymbol{A} \vee \boldsymbol{B}) = (\boldsymbol{B} \vee \boldsymbol{A})$
- Identity
  - $(\boldsymbol{A} \wedge \boldsymbol{T}) = \boldsymbol{A}$
  - $(\boldsymbol{A} \vee \boldsymbol{T}) = \boldsymbol{T}$
  - $(\boldsymbol{A} \wedge \boldsymbol{F}) = \boldsymbol{F}$
  - $(\boldsymbol{A} \vee \boldsymbol{F}) = \boldsymbol{A}$

Boolean Axioms

- Idempotent
  - $(\boldsymbol{A} \wedge \boldsymbol{A}) = \boldsymbol{A}$
  - $(\boldsymbol{A} \vee \boldsymbol{A}) = \boldsymbol{A}$
- Involution
  - $\overline{\overline{\boldsymbol{A}}} = \boldsymbol{A}$
- Complement
  - $(\boldsymbol{A} \wedge \overline{\boldsymbol{A}}) = \boldsymbol{F}$
  - $(\boldsymbol{A} \vee \overline{\boldsymbol{A}}) = \boldsymbol{T}$

Boolean Axioms

- Associativity
  - $(\boldsymbol{A} \wedge \boldsymbol{B}) \wedge \boldsymbol{C} = \boldsymbol{A} \wedge (\boldsymbol{B} \wedge \boldsymbol{C})$
  - $(\boldsymbol{A} \vee \boldsymbol{B}) \vee \boldsymbol{C} = \boldsymbol{A} \vee (\boldsymbol{B} \vee \boldsymbol{C})$
- Distributivity
  - $\boldsymbol{A} \wedge (\boldsymbol{B} \vee \boldsymbol{C}) = (\boldsymbol{A} \wedge \boldsymbol{B}) \vee (\boldsymbol{A} \wedge \boldsymbol{C})$
  - $\boldsymbol{A} \vee (\boldsymbol{B} \wedge \boldsymbol{C}) = (\boldsymbol{A} \vee \boldsymbol{B}) \wedge (\boldsymbol{A} \vee \boldsymbol{C})$

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Proving Boolean Equivalence

- Using the Boolean axioms (Example):
  - prove that: $(A \wedge B) \vee (\sim A \wedge C) \vee (B \wedge C) = (A \wedge B) \vee (\sim A \wedge C)$
  - LHS = $(A \wedge B) \vee (\sim A \wedge C) \vee (B \wedge C) = (A \wedge B) \vee (\sim A \wedge C) \vee [(B \wedge C) \wedge \boldsymbol{T}]$
    $= (A \wedge B) \vee (\sim A \wedge C) \vee [(B \wedge C) \wedge (A \vee \sim A)]$
    $= (A \wedge B) \vee (\sim A \wedge C) \vee [(B \wedge C \wedge A) \vee (B \wedge C \wedge \sim A)]$
    $= [(A \wedge B) \vee (B \wedge C \wedge A)] \vee [(\sim A \wedge C) \vee (B \wedge C \wedge \sim A)]$
    $= [(A \wedge B) \wedge (\boldsymbol{T} \vee C)] \vee [(\sim A \wedge C) \wedge (\boldsymbol{T} \vee B)]$
    $= [(A \wedge B)] \vee [(\sim A \wedge C)]$ = RHS
  - $\boldsymbol{T}$ means always TRUE.

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Read more FROM
John Kelly, The Essence of Logic (Prentice Hall, 1997)
Chapter 1 is good enough

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